1. (10%) Let
\[ f(x, y) = \begin{cases} 
\frac{2xy^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0); \\
0 & \text{if } (x, y) = (0, 0).
\end{cases} \]
Is \( f \) continuous at \((0, 0)\)? Justify your answer.

2. (5 %) Suppose that \( f \) is a differentiable function on \( \mathbb{R}^2 \) and \( f_x(x, y) = 2x + 3y \) and \( f_y(x, y) = 1 + 3x \). Let \( z = f(u, u + v) \). Find \( \partial z/\partial u \).

3. (10%) Let \( f(x) = \int_0^x e^{-t^2} \, dt \) for \( x \in (-\infty, \infty) \) and \( g(u, v) = f(uv) \) for \( (u, v) \in \mathbb{R}^2 \). Find \( \partial g(u, v)/\partial u \).

4. (20 %) Let \( f(x) = \frac{\ln(x)}{x} \) for \( x > 0 \).
   (a) Show that there exists a positive integer \( m \) such that \( f \) is decreasing on \([m, \infty)\).
   (b) Show that \( \sum_{k=0}^{\infty} (-1)^k f(k) \) converges conditionally.

5. (15%) Let \( f(x) = e^x + \ln(1 + x) \) for \( x > -1 \). Let \( r \) be the radius of convergence of the Maclaurin series of \( f(x) \).
   (a) Find the Maclaurin series of \( f(x) \).
   (b) Find \( r \).
   (c) Show that \( f(x) \) is equal to its Maclaurin series for \( |x| < r \).

6. (5 %) Show that \( \lim_{n \to \infty} \frac{x^n}{n!} = 0 \) for every \( x \in (-\infty, \infty) \).

7. (20%) Find the following integrals.
   (a) \( \int_0^\infty x e^{-x^2} \, dx \).
   (b) \( \int_0^{2\pi} x \sin(x) \, dx \).

8. (15%) Suppose that \( y = f(x) \) and the function \( f \) is determined by the equation
   \[ 2y \cos(x) + \sin(2y) = 0 \]
   when \( (x - \frac{\pi}{2})^2 + (y - \pi)^2 < 0.01 \). Can we conclude that \( f \) has a local minimum or local maximum at \( \pi/2 \)? Justify your answer.
1 Multiple Choice (2.5 points each)

Identify the letter of the choice that best completes the statement or answers the question.

1. When a country allows trade and becomes an importer of a good, which of the following would NOT be true?
   A. The gains of domestic consumers exceed the losses of domestic producers.
   B. The losses of domestic producers exceed the gains of domestic consumers.
   C. The price paid by domestic consumers of the good decreases.
   D. The price received by domestic producers of the good decreases.

2. The slope of the total product curve reveals information about the
   A. average product of workers.
   B. marginal product of workers.
   C. total product of workers.
   D. fixed product of workers.

3. If marginal cost exceeds marginal revenue
   A. the firm must be experiencing losses.
   B. the firm may still be earning a profit.
   C. the firm is most likely to be at a profit maximizing level of output.
   D. a profit maximizing firm should always increase the level of production.

4. A market force that can prevent firms from price discriminating is
   A. arbitrage.
   B. fluctuating resource prices.
   C. high fixed costs.
   D. All of the above.

5. As a group, oligopolists would always be better off if they would act collectively
   A. as a single monopolist.
   B. as a single competitor.
   C. as if they were each seeking to maximize their own profit.
   D. in a manner that would prohibit collusive agreements.

6. If an increase in the interest rate lowers savings, then
   A. the substitution effect is greater than the income effect.
   B. the income effect is greater than the substitution effect.
   C. the income effect and the substitution effect move in the same direction.
   D. we are unable to determine the sizes of the income and substitution effects without more information.
7. Quality Motors is a Japanese-owned company that produces automobiles; all of its automobiles are produced in American plants. In 2008, Quality Motors produced $25 million worth of automobiles and sold $12 million in the U.S. and $13 million in Mexico. In addition, it sold $2 million from the previous year's inventory in the U.S. The transactions just described contribute how much to U.S. GDP for 2008?
   A. $12 million.
   B. $14 million.
   C. $25 million.
   D. $27 million.

8. George lived in a home that was newly constructed in 2005 for which he paid $200,000. In 2008 he sold the house for $225,000. Which of the following statements is correct regarding the sale of the house?
   A. The 2008 sale increased 2008 GDP by $225,000 and had no effect on 2005 GDP.
   B. The 2008 sale increased 2008 GDP by $25,000 and had no effect on 2005 GDP.
   C. The 2008 sale increased 2008 GDP by $225,000; furthermore, the 2008 sale caused 2005 GDP to be revised upward by $25,000.
   D. The 2008 sale affected neither 2008 GDP nor 2005 GDP.

9. If marijuana were legalized, it is likely that there would be an increase in the supply of marijuana. Advocates of marijuana legalization argue that this would significantly reduce the amount of revenue going to the criminal organizations that currently supply marijuana. These advocates believe that the
   A. supply for marijuana is elastic.
   B. supply for marijuana is inelastic.
   C. demand for marijuana is elastic.
   D. demand for marijuana is inelastic.

10. If the price of Spanish olives imported into the United States decreases, then
    A. both the GDP deflator and the consumer price index will decrease.
    B. neither the GDP deflator nor the consumer price index will decrease.
    C. the GDP deflator will decrease, but the consumer price index will not decrease.
    D. the consumer price index will decrease, but the GDP deflator will not decrease.

11. If the natural rate of unemployment is 5.2 percent and the actual rate of unemployment is 5.7 percent, then by definition there is
    A. cyclical unemployment amounting to 0.5 percent of the labor force.
    B. frictional unemployment amounting to 0.5 percent of the labor force.
    C. structural unemployment amounting to 0.5 percent of the labor force.
12. If, at a given real interest rate, desired national saving were $120 billion, domestic investment were $85 billion, and net capital outflow were $45 billion. At that real interest rate in the loanable funds market, there would be a
   A. surplus; the real interest rate would rise.
   B. surplus; the real interest rate would fall.
   C. shortage; the real interest rate would rise.
   D. shortage; the real interest rate would fall.

13. Take the following information as given for a small, imaginary economy: consumption spending is $4,880 when income is $8,000; consumption spending is $5,200 when income is $8,400. Assume no investment accelerator nor any crowding-out effect, what is the government expenditure multiplier for this economy?
   A. 5.00.
   B. 5.25.
   C. 6.00.
   D. 6.25.

14. According to the ‘paradox of thrift,’ increased efforts to save will cause
   A. an increase in income and an increase in overall saving.
   B. an increase in income but no overall change in saving.
   C. a decrease in income but an increase in saving.
   D. a decrease in income and an overall decrease in saving.

15. In the U.S. an iPhone costs $700. The same iPhone in London sells for 500 pounds. If the nominal exchange rate were .63 pounds per dollar, then which of the following would be correct?
   A. The real exchange rate is greater than 1. A person in London with $700 could exchange them for pounds and have more than enough to buy the iPhone there.
   B. The real exchange rate is greater than 1. A person in London with $700 could exchange them for pounds but then wouldn’t have enough to buy the iPhone there.
   C. The real exchange rate is less than 1. A person in London with $700 could exchange them for pounds and have more than enough to buy the iPhone there.
   D. The real exchange rate is less than 1. A person in London with $700 could exchange them for pounds but then wouldn’t have enough to buy the iPhone there.

16. Changes in nominal variables are determined mostly by the quantity of money and the monetary system according to
   A. the classical dichotomy, but not the quantity theory of money.
   B. both the classical dichotomy and the quantity theory of money.

備

一、作答於試題上者，不予計分。
二、試題請隨卷繳交。
C. the quantity theory of money, but not the classical dichotomy.
D. neither the classical dichotomy nor the quantity theory of money.

2 Problems

1. Consider a monopolistically competitive market. Each firm faces a downward sloping demand function and incurs a fixed entry cost. The marginal cost is increasing. The average cost is U-shaped. There are no barriers to entry and exit. Answer the following questions.
   a. (10 points) Draw the inverse demand, marginal revenue, average cost, and marginal cost functions to show the long-run equilibrium.
   b. (10 points) At long-run equilibrium, do all firms produce at the minimum of the average cost? Explain.

2. Isoland is a small country that produces and consumes jelly beans. The world price of jelly beans is \$2 per bag, and Isoland’s domestic demand and supply for jelly beans are governed by the following equations:
   Demand: \( Q^D = 16 - P \)
   Supply: \( Q^S = P \),
where \( P \) is in dollars per bag and \( Q \) is in bags of jelly beans.
   a. (8 points) Suppose there are no trade restrictions. Calculate the equilibrium price, quantities of imports, consumer surplus, and producer surplus.
   b. (8 points) Now suppose the Czar of Isoland responds to the pleas of jelly bean producers by placing a \$2 per bag tariff on jelly bean imports. Calculate the equilibrium price, quantities of imports, government revenue, and the deadweight loss.
   c. (4 points) Suppose, instead of the tariff, the Czar places a \$2 per bag tax on jelly bean consumption (including both imported and domestically produced jelly beans). Calculate the tax revenue for the government, and the deadweight loss.

3. a. (4 points) What are expansionary fiscal policy and expansionary monetary policy?
   b. (10 points) Why doesn’t fiscal policy always work perfectly in smoothing out business cycles?
   c. (6 points) Why doesn’t monetary policy always work perfectly in helping the economy to recover from recessions?

備註
一、作答於試題上者，不予計分。
二、試題請隨卷繳交。