Part I. True or False. 16 Points. Answer T for true or F for false in each of the following questions.

1. Activation records of function calls in modular programming are stored in queues rather than in stacks.
2. Hashing is one of the most efficient types of searching and is supported by a good hash function, which is to approximate uniform hashing.
3. The computational complexity of inserting a node into a heap is $O(n \log n)$.
4. Breath-first search is more useful than depth-first search in finding minimum spanning trees.
5. The balancing factor of a node in an AVL tree is an integer that can stay in a range from -2 to +2.
6. Hoffman coding performs effective compression regardless of the frequency distribution of symbols in the data.
7. The worst case analysis gives us a lower bound on the performance of an algorithm.
8. Quick sort uses principles of probability to virtually guarantee average-case performance.

Part II. Single Selection. 24 Points.

1. O-notation reflects an algorithm’s order of (a) accuracy (b) growth (c) power (d) size in the runtime complexity.
2. The computational complexity of generating all possible permutations of a set of data is (a) $O(n \log n)$ (b) $O(n^2)$ (c) $O(2^n)$ (d) $O(n!)$
3. Which of the stack operations requires a runtime complexity of $O(n)$, as opposed to $O(1)$? (a) initialize a stack (b) remove all elements in a stack (c) push an element into a stack (d) pop an element off a stack
4. Which traversing order is best suited for appropriately removing a sub-tree in a binary tree? (a) pre-order (b) in-order (c) post-order (d) level-order
5. In hashing, secondary clustering occurs because (a) many keys hash to the same location (b) too many items with the same key are inserted (c) the hash function is not perfect (d) the sequence of step length is always the same
6. Which of the following statements about graph as a data structure is NOT correct? (a) If removing a vertex disconnects a graph or component, the vertex is a bridge. (b) A directed graph is strongly connected if every vertex is reachable from each other by following some path. (c) Incidence is a relation between a vertex and an edge. (d) Adjacency is a relation between two vertices.
7. Suppose we need to sort all of the customer records by name for a top-tier international corporation. The size of the data is so large that it cannot be loaded into memory all at once. Which sorting algorithm should we use? (a) heap sort (b) insertion sort (c) merge sort (d) quick sort
8. Dijkstra’s algorithm finds the shortest path (a) from one specified vertex to another specified vertex (b) from one specified vertex to all other vertices (c) from all vertices to all other vertices that can be reached along one edge (d) from all vertices to all other vertices that can be reached along multiple edges.
Part III. Short Answer Questions. 60 Points.

1. (10 points) Consider the two types of data structures - linked list and array. (a) Give some usage contexts when each type of data structure is preferable than the other. (b) To support virtual memory, pages of virtual address space must be mapped to frames in physical memory. Which data structure, linked list or array, will you use for frame allocation management? Why?

2. (10 points) A system receives five requests, req1, req2, req3, req4, req5, in order, and wishes to sequentially process req1, req3, and req5, while leaving req2 and req4 in the queuing order. (a) Suppose a queue is used to store the requests, what is the appropriate sequence of queue operations? (b) Suppose a linked list is used to store the requests, what is the appropriate sequence of linked list operations?

3. (16 points) Consider using a binary tree to process an arithmetic expression as an expression tree that contains two types of objects: operators and terminal values. The sub-trees rooted at the children of each node are the operands of the operator stored in the parent. Given the arithmetic expression, \(((87 - 18)/23) \times (15 + 10))\). (a) Draw the expression tree. (b) Give the prefix expression by traversing the tree in pre-order. (c) Give the postfix expression by traversing the tree in post-order. (d) Which traversing order is better suited to process by using an abstract stack? Show the sequence of stack operations to derive the value of the expression.

4. (12 points) Given the following recursive definition:

\[
H(n) = \begin{cases} 
1, & \text{if } n = 1 \\
H(n - 1) + \left(\frac{1}{n}\right), & \text{if } n > 1 
\end{cases}
\]

(a) Show the computation of the function \(H(n)\) in expanded expression?
(b) A recursive function is tail recursive if all recursive calls within it are tail recursive. A recursive call is tail recursive when it is the last statement that will be executed within the body of a function and its return value is not a part of an expression. Is \(H(n)\) tail recursive? If yes, why? If not, how can it be modified into a tail recursive version?
(c) What is the advantage of a tail recursive function in execution?
5. (12 points) A double-ended list is a linked list with both references to the first element and the last element. A doubly-linked list is composed of elements linked in both directions.

(a) What is the advantage of a double-ended list over a singly-linked list? What is the advantage of a doubly-linked list over a singly-linked list?

Given the following doubly-linked list definition:

```c
typedef struct DList_Elem {
    void        *data;
    struct DList_Elem *prev;
    struct DList_Elem *next;
} DList_Elem;

typedef struct DList {
    int         size;
    DList_Elem *head;
    DList_Elem *tail;
} DList;

int dlist_ins_after (DList *list, DList_Elem *element, const void *data);
int dlist_del (DList *list, DList_Elem *element);
#define dlist_size (list) ((list)->size)
```

(b) Write a C program to implement the operation, dlist_ins_after, that inserts an element into a doubly-linked list right after a specified element.

(c) Write a C program to implement the operation, dlist_del, that deletes a specified element from a doubly-linked list.
第一題[48%]: 單一選擇題，每小題答對得六分，未答得零分，答錯倒扣兩分。

1.1 Which of the following statements is wrong regarding the Interprocess Communication?

(A) Message passing is easier to implement in a distributed system than shared memory.

(B) For direct communication, a link is established automatically between every pair of processes that want to communicate.

(C) For indirect communication, between each pair of communicating processes, there may be a number of different links, with each link corresponding to one mailbox.

(D) As for the message passing systems, “non-blocking send” can be used in the case of “zero capacity” buffering.

1.2 The following processes are being scheduled using a preemptive, round-robin scheduling algorithm. Each process is assigned a numerical priority, with a higher number indicating a higher relative priority. In addition to the processes listed below, the system also has an idle task (which consumes no CPU resources and is identified as P_idle). This task has priority 0 and is scheduled whenever the system has no other available processes to run. The length of a time quantum is 10 units. If a process is preempted by a higher-priority process, the preempted process is placed at the end of the queue.

<table>
<thead>
<tr>
<th>Process</th>
<th>Priority</th>
<th>Burst Time</th>
<th>Arrival Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>40</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>30</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>P3</td>
<td>30</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>P4</td>
<td>35</td>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td>P5</td>
<td>5</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>P6</td>
<td>10</td>
<td>10</td>
<td>105</td>
</tr>
</tbody>
</table>

Which of the following statements is wrong?

(A) The turnaround time for P1 is 20.

(B) The turnaround time for P2 is 50.

(C) The waiting time for P3 is 35.

(D) The waiting time for P4 is 0.

1.3 Consider a computer system with a 32-bit logical address and 4-KB page size. The system supports up to 512 MB of physical memory. How many entries are there in a conventional single-level page table and an inverted page table, respectively?

(A) $2^{20}$, $2^{17}$

(B) $4K$, $512M$

(C) $2^{32}$, $2^{29}$

(D) $2^8$, $2^8$
1.4 The rate-monotonic scheduling algorithm assumes that the processing time of a periodic process is the same for each CPU burst. Let's consider two processes, P1 and P2. The periods for P1 and P2 are 50 and 75, respectively. The processing times are 25 for P1 and 30 for P2. The deadline for each process requires that it completes its CPU burst by the start of its next period.

Which one of the following is correct?

(A) The CPU utilization of P1 is 25/50 = 0.5  
(C) Using the EDF (Earliest Deadline First) scheduling, both P1 and P2 can meet their deadlines.

(B) Using the rate-monotonic scheduling, either P1 or P2 cannot meet its deadline.

1.5 Regarding semaphore, which one of the followings is correct?

(A) To guarantee that no two processes can execute wait() and signal() on the same semaphore at the same time, we can use a priority-inheritance protocol to solve the priority inversion problem.

(B) To ensure that wait() and signal() are performed atomically, it is efficient that we can simply inhibit interrupts on every processor in a multiprocessor environment.

1.6 Consider a file system on a disk that has both logical and physical block size of 512 bytes. Assume that the information about each file is already in memory. If we are currently at logical block 10 (the last block accessed was block 10) and want to access logical block 4, how many physical blocks must be read from the disk for each of the three allocation strategies (contiguous, liked, and indexed), respectively?

(A) (1, 4, 2)  
(C) (2, 4, 1)

(B) (1, 1, 1)  
(D) (4, 2, 2)
1.7 Consider a system with 5 processes \( P_0 \) through \( P_4 \) and 4 resource types A, B, C, and D. Suppose that the system has the following resource allocation status:

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C D</td>
<td>A B C D</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>2 0 0 1</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>3 1 2 1</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>2 1 0 3</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>1 3 1 2</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>1 4 3 2</td>
</tr>
</tbody>
</table>

Using the Banker’s algorithm, which one of the following statements is incorrect?

(A) The matrix \( \text{Need} \) is

<table>
<thead>
<tr>
<th>( P_0 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C D</td>
<td>A B C D</td>
<td>A B C D</td>
<td>A B C D</td>
<td>A B C D</td>
</tr>
<tr>
<td>2 2 1 1</td>
<td>2 1 3 1</td>
<td>0 2 1 3</td>
<td>0 1 1 2</td>
<td>2 2 3 3</td>
</tr>
</tbody>
</table>

(C) If a request from process \( P_4 \) arrives for \( (0,0,2,0) \), the request can be granted immediately.

(B) The system is in a safe state.

(D) If a request from process \( P_1 \) arrives for \( (1,1,0,0) \), the request can be granted immediately.

1.8 Assume that a program has just referenced an address in virtual memory. Which one of the following scenarios can NOT occur?

(A) TLB miss with no page fault
(B) TLB miss and page fault
(C) TLB hit and no page fault
(D) TLB hit and page fault

第二題 [12%]

In a real computer system, neither the resources available nor the demands of processes for resources are consistent over long periods (months). Resources break or are replaced, new processes come and go, new resources are bought and added to the system. If deadlock is controlled by the banker’s algorithm, which of the following changes can be made safely (without introducing the possibility of deadlock), and under what circumstances?

(a) Increase \( \text{Available} \) (new resources added)
(b) Increase \( \text{Max} \) for one process (the process needs or wants more resources than allowed)
(c) Increase the number of processes
<table>
<thead>
<tr>
<th>考試科目</th>
<th>作業系統</th>
<th>所別</th>
<th>資訊科學系</th>
<th>考試時間</th>
<th>2月23日(日) 第二節</th>
</tr>
</thead>
</table>

第三題 [8%]
(a) Consider the following code segment:

```c
pid_t pid;

pid = fork();
if(pid == 0) { /* child process */
    fork();
    thread_create(...);
}
fork();
```

How many unique processes are created? [2%]
How many unique threads are created? [2%]

(b) A system with two dual-core processors has four processors available for scheduling. A CPU-intensive application is running on this system. All input is performed at program start-up, when a single file must be opened. Similarly, all output is performed just before the program terminates, when the program results must be written to a single file. Between startup and termination, the program is entirely CPU-bound. Your task is to improve the performance of this application by multi-threading it. The application runs on a system that uses the one-to-one threading model (each user thread maps to a kernel thread). How many threads will you create to perform the input and output efficiently? Explain. [2%]
How many threads will you create for the CPU-intensive portion of the application efficiently? Explain. [2%]

第四題 [10%]
Suppose that a disk drive has 5,000 cylinders, numbered 0 to 4999. The drive is currently serving a request at cylinder 2150, and the previous request was at cylinder 1805. The queue of pending requests, in FIFO order, is:

2069, 1212, 2296, 2800, 544, 1618, 356, 1523, 4965, 3681

Starting from the current head position, what is the total distance (in cylinders) that the disk arm moves to satisfy all the pending requests for each of the following disk-scheduling algorithms?
(a) SSTF
(b) C-SCAN
第五题 [10%]
Consider the following page reference string:
7, 2, 3, 1, 2, 5, 3, 4, 6, 7, 7, 1, 0, 5, 4, 6, 2, 3, 0, 1
Assuming demand paging with three frames allocated for a process in a virtual memory system, how many page faults would occur for the following replacement algorithms?
(a) Optimal
(b) Second-chance

第六题 [12%]
Suppose we have \( n \) processes, \( P_0 \sim P_{n-1} \). The common data structures are:

```java
boolean waiting[n];
boolean lock;
```

The global variable `lock` is initialized to false. All the `waiting[i]` are initialized to false.

The following is the structure for process \( P_i \):

```java
do {
    waiting[i] = true;
    key = true;
    while(waiting[i] && key)
        key = test_and_set(&lock);
    waiting[i] = false;
    /*critical section*/
    j = (i+1)%n;
    while ((j!=i) && !waiting[j])
        j = (j+1)%n;

    if (j==i)
        lock = false;
    else
        waiting[j] = false;
    /* remainder section*/
}while(true);
```

Does the above algorithm satisfy all three requirements (mutual exclusion, progress, bounded waiting)? Justify your answer.
1. (7%) (a) (3%) Find the least integer $n$ such that $f(x)$ is $O(x^n)$ when
\[ f(x) = 2x^2 + x^3 \log x + (x^5 + x^2 + 1)/(x^5 + 1). \]
(b) (4%) Give as good a big-O estimate as possible for
\[ (n \log n + 1)^2 + n(2^n) + n^{(n^2)} + (\log n + 1)(n^2 + 1) \]

2. (10%) Determine whether each of the following equations are linear homogeneous recurrence relations with constant coefficients or not. Explain your answer if your answer is not.
   a) $a_n = 7a_{n-1} + 5a_{n-2} + 1$
   b) $a_n = (n - 1)a_{n-1} + a_{n-2}$
   c) $f_n = (1.22)f_{n-25} + 3.75f_{n-30}$
   d) $f_n = f_{n-1}^2 + f_{n-2}$
   e) $f_n = 2^n + 3^{n-1}$

3. (13%) (a) (7%) Compute $s$ where $s=3^{171} \mod 143$ (hint: use the Chinese Remainder Theorem)
   (b) (6%) Find the least positive integer $x$ such that $13x \equiv 1 \pmod{2436}$

4. (10%) Find the least number of cables required to connect 25 computers to 15 printers to guarantee that at any time any set of 15 or fewer computers can simultaneously access different printers via direct connections. Justify your answer.

5. (10%) Determine whether the relations represented by these zero-one matrices are equivalence relations or not. Justify your answer.
   \[
   (a) \begin{pmatrix}
   1 & 1 & 1 \\
   1 & 1 & 1 \\
   1 & 0 & 1 \\
   0 & 1 & 1 \\
   \end{pmatrix}
   \]
   \[
   (b) \begin{pmatrix}
   1 & 1 & 1 \\
   1 & 1 & 1 \\
   1 & 1 & 1 \\
   0 & 0 & 1 \\
   \end{pmatrix}
   \]

6. (10%) Define $h(T)$ the height of a fully binary tree $T$ and $n(T)$ the number of vertices in the tree. Prove that $n(T) \leq 2^{h(T)+1} - 1$. 
7. (10%) Please find the inverse of each of the following matrices (if it exists):

(a) \[
\begin{bmatrix}
1 & 2 & 3 \\
2 & 6 & 1 \\
3 & 10 & -1
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 3 & -2 \\
2 & 8 & -3 \\
1 & 7 & 1
\end{bmatrix}
\]

8. (10%) Given the following functions, please determine each of the following function is linear or non-linear.

(a) \( F : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) defined by \( F(x, y, z) = (x + y + z, 2x - 3y + 4z) \)

(b) \( F : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) defined by \( F(x, y) = (xy, x) \)

(c) \( F : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) defined by \( F(x, y) = (x + 3, 2y, x + y) \)

(d) \( F : \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) defined by \( F(x, y, z) = (x + 1, y + z) \)

9. (10%) Please compute the determinant of the following matrix:

\[
\begin{vmatrix}
6 & 2 & 1 & 0 & 5 \\
2 & 1 & 1 & -2 & 1 \\
1 & 1 & 2 & -2 & 3 \\
3 & 0 & 2 & 3 & -1 \\
-1 & -1 & -3 & 4 & 2
\end{vmatrix}
\]

10. (10%) Let \( A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix} \)

(a) Please find eigenvalues and corresponding eigenvectors

(b) Please find a nonsingular matrix \( P \) such that \( D = P^{-1}AP \) is diagonal

(c) Please find \( A^6 \) and \( f(A) \) where \( f(t) = t^4 - 5t^3 + 7t^2 - 2t + 5 \)

(d) Please find a matrix \( B \) such that \( B^2 = A \)